Bound on the time variation of the fine structure constant driven by quintessence

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The bound on the time variation of the fine structure constant (α) driven by the dynamics of quintessence scalar field which is coupled to electromagnetism is discussed using phenomenological quintessential models constrained by SNIa and CMB observations. We find that those models allowing early quintessence give the largest variation $\Delta\alpha$ at the decoupling epoch. Furthermore, the fifth force experiments imply that $\Delta\alpha/\alpha$ is less than about 0.1%.

The idea of time-varying fundamental physical constants such as gauge coupling constants was proposed long time ago [1]. Recently it has been claimed that the results of a search for time variability of the fine structure constant (α), using absorption systems in the spectra of distant quasars, yield 5-σ evidence for a smaller α in the past: $\Delta\alpha/\alpha = -0.543 \pm 0.116 \times 10^{-5}$ over the redshift range $0.2 < z < 3.7$ [2]. In addition, there exist terrestrial and cosmological constraints. An analysis of the isotropic abundances in the Oklo natural uranium fission reactor, active about 1.8×10^9 years ago (corresponding to $z \simeq 0.14$), suggests $-0.9 \times 10^{-7} < \Delta\alpha/\alpha < 1.2 \times 10^{-7}$ [3]. A bound $\Delta\alpha/\alpha < 3 \times 10^{-7}$ has been obtained from the analysis of the Re/Os ratio in meteorites ($z \simeq 0.45$) [4]. While the data of cosmic microwave background (CMB) anisotropies are consistent with α being smaller by a few percent at the decoupling epoch ($z \sim 1100$) in a flat cold dark matter model with a cosmological constant (ΛCDM) [5], the recent observations made by the Wilkinson Microwave Anisotropy Probe (WMAP) have provided a bound $-0.06 < \Delta\alpha/\alpha < 0.02$ at 95% CL [6]. At much higher redshifts, big bang nucleosynthesis considerations place bounds $|\Delta\alpha/\alpha| < 10^{-2}$ at $z \sim 10^9$ [7].

The recent astrophysical and cosmological observations such as type Ia supernovae (SNe) and CMB anisotropies concordantly prevail the ΛCDM model [8]. The current data, however, are consistent with a somewhat broader diversity of such "dark energy" as long as its equation of state (EOS) approaches that of the cosmological constant at a recent epoch. A dynamically evolving scalar field ϕ called "quintessence" (Q) is probably the most popular scenario so far to accommodate the dark energy component. Many Q models have been proposed with various physically motivated effective potentials $V(\phi)$ for the scalar field [9]. Several attempts have been made to test different Q models [10]. Nevertheless, it proves to be premature at this stage to perform a meaningful data fitting to a particular Q model, or to differentiate between the variations. The reconstruction of $V(\phi)$ would likely require next-generation observations. As such, model independent approaches which simply involve parametrizing the dark energy EOS have been proposed to comply with the SNe and CMB observational data [11, 12, 13].

It is of particular interest to study the observational effects of direct interaction of ϕ to ordinary matter if there is any. For example, imposing an approximate global symmetry would allow a coupling of ϕ to electromagnetism, which would lead to rotation of polarized light from distant radio sources, temporal evolution of α [14], and generation of primordial magnetic fields [15]. Recently, there have been many studies on the time-varying α in the context of quintessential cosmology by invoking non-renormalizable ϕ-photon couplings [16, 17, 18]. However, most of the studies are based on certain Q models [17]. In this paper, we will discuss the observational constraints to the evolution of ϕ through its EOS $p_\phi = w_\phi \rho_\phi$ in the parametrized Q models [11, 12, 13] that in turn give rise to a model independent bound on the Q-driven time variation of α at the decoupling epoch.

The ϕ-photon coupling that we consider here is

$$L_{\phi\gamma} = -\frac{\kappa}{4}\frac{\phi}{M_p}g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, M_p is the reduced Planck mass $(8\pi G)^{-1/2}$, and κ is a coupling constant. The fifth-force experiments limit [19]

$$\kappa < 10^{-3}. \tag{2}$$

Therefore, by defining $\theta \equiv \phi/M_p$, the change of α from the present time to a time t is simply given by

$$\frac{\Delta\alpha}{\kappa\alpha} = \theta_0 - \theta(t). \tag{3}$$

Consider a flat universe in which the total density parameter of the Universe today is represented by $\Omega_0 = \Omega_m^0 + \Omega_r^0 + \Omega_\phi^0 = 1$ with $\Omega_m^0 h^2 = 0.135$ and $\Omega_r^0 h^2 = 4.152 \times 10^{-5}$, where the present Hubble constant is parametrized as $H_0 = 100h\,\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$. Assuming a spatially homogeneous ϕ field, the evolution of the cosmic background is governed by

$$d\varrho_\phi/d\eta = -3a\mathcal{H}(1 + w_\phi)\varrho_\phi, \tag{4}$$

$$d\mathcal{H}/d\eta = -\frac{3}{2}a\mathcal{H}^2 - \frac{1}{2}a(w_r\varrho_r + w_\phi\varrho_\phi), \tag{5}$$

where a is the scale factor and the conformal time is defined as $\eta = H_0 \int dt a^{-1}(t)$. Here we have used $(d\phi/dt)^2 = (1 + w_\phi)\rho_\phi$ and $V(\phi) = (1 - w_\phi)\rho_\phi/2$, and rescaled the energy density of the ith component as $\varrho_i \equiv \rho_i/(M_p H_0)^2$. Accordingly, the dimensionless Hubble parameter is given by

$$\mathcal{H}^2 \equiv (H/H_0)^2 = \Omega_m^0 \, a^{-3} + \Omega_r^0 \, a^{-4} + \Omega_\phi \, \mathcal{H}^2. \quad (6)$$

Therefore, given the prescribed EOS $w_\phi(\eta)$ which is a function of conformal time, the problem is reduced to solving a set of first-order coupled ordinary differential equations (4) and (5) with the initial conditions set at the present time and the time variable running backward. Let us introduce a useful quantity that is the Ω_ϕ-weighted average [20]

$$\langle w_\phi \rangle \equiv \int_{\eta_{\rm dec}}^{\eta_0} \Omega_\phi(\eta) w_\phi(\eta) d\eta \times \left(\int_{\eta_{\rm dec}}^{\eta_0} \Omega_\phi(\eta) d\eta \right)^{-1}, \quad (7)$$

where η_0 and $\eta_{\rm dec}$ are the conformal time today and at decoupling respectively. It is known that $w_\phi \neq 0$ after decoupling would result in a time-varying Newtonian potential which produces large-scale CMB anisotropy through the integrated Sachs-Wolfe (ISW) effect. It was shown [20, 21] that as far as the CMB anisotropy power spectrum is concerned, the time-averaged $\langle w_\phi \rangle$ can be well approximated by an effective constant value for w_ϕ as long as the Q component is negligible at decoupling.

Physically, $-1 \leq w_\phi \leq 1$, where the former equality holds for a pure vacuum state. Lately some progress has been made in constraining the behavior of ϕ from observational data. A combined large scale structure, SNe, and CMB analysis has set an upper limit on Q models with a constant $w_\phi < -0.7$ [21, 22], and the recent WMAP CMB data gives a stronger limit $w_\phi < -0.78$ [23]. Furthermore, the SNe data and measurements of the position of the acoustic peaks in the CMB anisotropy spectrum have been used to put a constraint on the present $w_\phi^0 \leq -0.96$ [24]. The apparent brightness of the farthest SN observed to date, SN 1997ff at redshift $z \sim 1.7$, is consistent with that expected in the decelerating phase of the flat ΛCDM model with $\Omega_\Lambda \sim 0.7$ [25], implying $w_\phi = -1$ for $z < 1.7$.

Let us first work on the simplest case with a constant $w_\phi = -0.8$ to obtain an order-of-magnitude estimate on $\Delta\alpha/\kappa\alpha$. By solving

$$d\theta/d\eta = a\sqrt{(1 + w_\phi)\varrho_\phi}, \quad (8)$$

we find that $\Delta\alpha/\kappa\alpha$ decreases quickly with redshift to a constant value of -0.82 at $z \sim 20$. Next we study the varying α in two phenomenological Q models which are based on the maximum likelihood analysis of the current SNe and CMB observational data. The first model has a monotonic EOS $w_\phi(a) = w_0 + (w_m - w_0)(1 - a)$ (where $w_\phi \to w_m$ as $a \to 0$) that has been shown to be able to describe the late-time ϕ evolution for a wide class of Q models [11]. Using $w_0 = -0.99$ and $w_m = -0.31$, which

are within 2σ from the best-fit parameters to the SNe and CMB data [26], we find that $\Delta\alpha/\kappa\alpha$ decreases to -1.63 at decoupling. The second model has an EOS with four parameters $W = (w_0, w_m, a_c, \Delta)$ [12]:

$$w_\phi(a) = w_0 + (w_m - w_0) \frac{\left(1 + e^{a_c/\Delta}\right)\left(1 - e^{(1-a)/\Delta}\right)}{\left(1 + e^{(a_c-a)/\Delta}\right)\left(1 - e^{1/\Delta}\right)}, \quad (9)$$

where a_c signifies the transition epoch and Δ represents the transition rate from w_0 to w_m. We have worked out two cases: a slow EOS with $W = (-1, -0.3, 0.1, 0.8)$ and a rapid EOS with $W = (-1, 0, 0.4, 0.1)$, both of which are taken from the 1σ values of the best-fit models to the WMAP and SNe data [27]. We find that $\Delta\alpha/\kappa\alpha$ decrease to -1.56 and -3.82 at decoupling for the slow and rapid EOS respectively. In fact, the slow EOS and the first model are very similar, in which Ω_ϕ/Ω_m decreases to 0.01 at $z \sim 30$ and becomes negligible in the dark ages. On the contrary, the rapid EOS which has a larger change in α allows early quintessence with $\Omega_\phi/\Omega_m \sim 0.1$ for $1100 > z > 10$. This can be understood by considering a constant w_ϕ for which the problem has a simple solution. Equation (4) gives $\varrho_\phi \propto a^{-3(1+w_\phi)}$ and $\Omega_\phi/\Omega_m \propto a^{-3w_\phi}$. Thus, from Eq. (8) we have $d\theta/d\eta \propto a^{-(1+3w_\phi)/2}$ and therefore the change of α will grow if $w_\phi \geq -1/3$. Also, the condition for early quintessence is simply $w_\phi \geq 0$. From the results of the above slow and rapid EOS's where $w_m = -0.3$ and $w_m = 0$ respectively, we see that the change of α in models with $0 > w_\phi > -1/3$ is smaller than that in models with early quintessence. Therefore, early quintessence is a crucial condition for a large temporal change of α. We thus see that within viable models the varying α strongly depends on the Q component in the dark ages.

To further study the Q dependence and to find out the maximum change of $|\Delta\alpha/\kappa\alpha|$, we turn to the generic quintessence (GQ) model [13], in which a piecewise constant EOS subject to observational constraints is introduced. The GQ model starts with determining the form for w_ϕ at low redshifts. We choose $w_\phi \simeq -1$ for $z \lesssim 2$ to satisfy the above-mentioned SNe constraints. Then w_ϕ should be increased to 1 after $z = 2$ to have a maximum change of α based on Eq. (8). However, to avoid an unacceptably large ISW effect, w_ϕ must drop to -1 at a certain redshift. This results in the square-wave EOS in the GQ model. A GQ model is shown in Fig. 1, where the width of the square wave is determined by $\langle w_\phi \rangle = -0.8$ to saturate the WMAP upper limit on a constant w_ϕ [23]. In this case, $\Delta\alpha/\kappa\alpha$ reaches a minimum value of -0.4. In fact, a square-wave or pulse-shaped EOS is anticipated and quite general in the class of Q models using pseudo Nambu-Goldstone boson fields incorporated with quantum corrections to the cosine potential [28]. However, one can still shift the whole square wave to higher redshifts while fixing $\langle w_\phi \rangle = -0.8$ as to generate the required ISW effect. We find that the shift increases the Q component at the decoupling epoch and affects the location of the CMB acoustic peaks. This can be understood as follows.
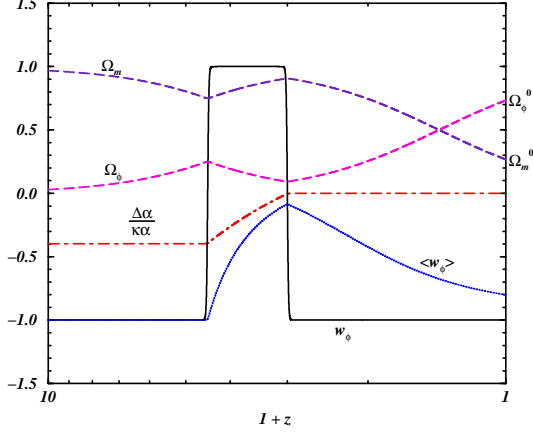
FIG. 1: $\Delta\alpha/\kappa\alpha$ in the GQ model with a square-wave EOS located at low redshift.
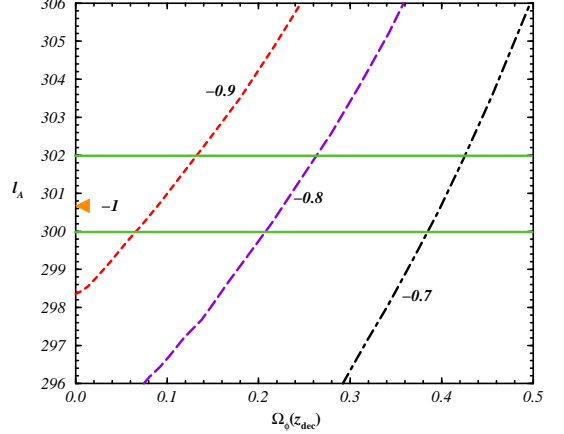

FIG. 2: Acoustic scales of the GQ models are plotted as a function of the quintessence density at decoupling while keeping $\langle w_\phi \rangle$ fixed. The model equivalent to the cosmological constant case has $l_A \simeq 301$ and is denoted by an arrow. The two horizontal lines signify the 1-σ upper and lower bounds permitted by the WMAP data [23].
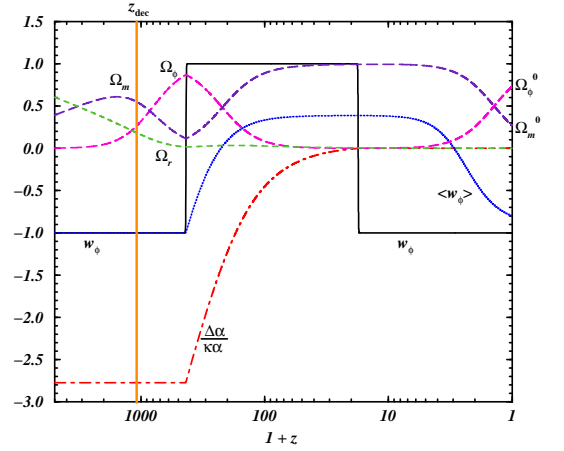
The tightly coupled baryon-photon plasma experienced a serial acoustic oscillation just before the recombination epoch. The acoustic scale (the angular momentum scale of the acoustic oscillation) sets the location of the peaks in the CMB anisotropy power spectrum [29], and is characterized by

$$l_A = \pi d_*/h_s = \pi(\eta_0 - \eta_{\mathrm{dec}})/\int_0^{\eta_{\mathrm{dec}}} c_s d\eta \; , \qquad (10)$$

where d_* represents the comoving distance to the decoupling epoch and h_s denotes the sound horizon at decoupling, both of which are affected in the presence of quintessence. The sound speed c_s in the pre-recombination plasma is given by [29]

$$c_s = 1/\sqrt{3(1+R)} \quad \text{with}$$
$$R \equiv \frac{3\rho_b}{4\rho_\gamma} \approx 30366 \left(\frac{T_\gamma^0}{2.725K} \right)^{-4} \frac{\Omega_b^0 h^2}{1+z} \; , \qquad (11)$$

where the baryon-photon momentum density ratio R sets the baryon loading to the acoustic oscillation of CMB. Hence, h_s at decoupling can be determined by the differential equation

$$dh_s/d\eta = c_s, \qquad (12)$$

coupled with the background evolution equations (4) and (5). Using $\Omega_b^0 h^2 = 0.0224$, $h = 0.71$, $T_\gamma^0 = 2.725K$, and $z_{\mathrm{dec}} = 1089$, we can calculate various acoustic scales by shifting the square wave of w_ϕ toward the decoupling epoch while keeping $\langle w_\phi \rangle$ fixed. Figure 2 plots the results against the Q field energy density at decoupling $\Omega_\phi^{\mathrm{dec}}$ with contours $\langle w_\phi \rangle = -0.7, -0.8, -0.9,$ and -1. The last one is simply the ΛCDM model. All GQ models lying in the region above the curve of $\langle w_\phi \rangle = -0.8$ and within the WMAP bounds are consistent with the current CMB data. Thus, the constant $w_\phi = -0.8$ model

FIG. 3: $\Delta\alpha/\kappa\alpha$ in the extreme GQ model with $(\langle w_\phi \rangle, \Omega_\phi^{\mathrm{dec}}) = (-0.8, 0.26)$. The decoupling epoch is marked as the vertical line at $z_{\mathrm{dec}} = 1089$.

and that in Fig. 1 are in fact disfavored as far as the acoustic peak location is concerned. In Fig. 3, we have worked out an extreme model with parameters taken at the upper right corner of the allowed region in Fig. 2. The model shows that the Q component dominates over the matter density at high redshifts. This early quintessence drives $\Delta\alpha/\kappa\alpha$ to a minimum value of -2.77. Note that a non-zero Ω_ϕ after decoupling would lead to a suppression of the growth of matter perturbations on scales smaller than the smoothing scale of the quintessence. Following Ref. [13], we have found that the growth function for mat-

ter perturbations in the extreme GQ model is still at an acceptable level with the COBE data. Furthermore, perturbations in the dark energy component will mildly affect our results because dark energy with $\langle w_\phi \rangle = -0.8$ is already close to a vacuum state [30]. At last, the extreme GQ model with $\Omega_\phi^{\text{dec}} = 0.26$ is consistent with the upper bound $\Omega_\phi < 0.39$ during the radiation dominated epoch obtained by performing a maximum likelihood analysis on the CMB data [31].

In conclusion we have investigated the quintessence-induced evolution of α allowed by the observational constraints from CMB and SNe using the parametrized quintessence models. Although the true equation of state, if there is any, may be a complicated function of time, they should capture the generic features of quintessence evolution for a model-independent study of the time-varying α. We have found an extreme GQ model and a rapid equation of state that allow early quintessence driving a maximum change of $|\Delta\alpha/\kappa\alpha| \sim 3 - 4$ at the decoupling epoch. Hence, the fifth-force limit in Eq. (2) implies that $|\Delta\alpha/\alpha|$ is less than about 0.1% at decoupling as long as the quintessence is coupled

to photon. Future CMB data will be able to constrain α up to 0.1% level or tighter than this when combined with other cosmological measurements [6].

So far, we have not considered using the phenomenological quintessence models to fit the observational data of varying α at low redshifts ($z < 3.7$). Although it is *ad hoc* and fine-tuned, we find that it is not so difficult to modify the equation of state at low redshifts to make the change of α consistent with the measurements of quasar absorption spectra and the Oklo and Re/Os limits. Indeed, using a model-independent approach it was shown that these low-redshift measurements can constrain the dark energy equation of state today to satisfy $-1 < w_\phi < -0.96$ and disfavor late-time changes in w_ϕ [18]. This result is consistent with what we have assumed for the low-redshift value of w_ϕ in our GQ models and the constraint $\Delta\alpha/\alpha = -0.06 \pm 0.06 \times 10^{-5}$ recently obtained based on a new sample of high-redshift ($0.4 \leq z \leq 2.3$) quasar absorption line systems [32].

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